Integrated Mathematics III

Adopted by the
Indiana State
Board of Education
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In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Integrated Mathematics III:

Standard 1 — Algebra and Functions

Students solve linear inequalities by using order properties of the real numbers and they operate with polynomials — adding, subtracting, multiplying, dividing, and raising to powers — and find factors of polynomials, learning special techniques for factoring quadratics. Students simplify algebraic fractions, using what they have learned about factoring polynomials. They solve algebraic proportions. They solve quadratic equations by using the formula, by factoring, and by completing the square. They also solve equations that contain radical expressions. Students recognize and graph polynomial, rational, and algebraic functions. They use a variety of methods to solve systems of up to three linear equations in up to three variables. Students write equations and draw graphs of conic sections (circle, ellipse, parabola, and hyperbola), thus relating an algebraic representation to a geometric one. They add, subtract, multiply, divide, and simplify algebraic fractions and solve equations involving algebraic fractions. Students understand the concepts of logarithmic and exponential functions. Students define the concepts of arithmetic and geometric sequences and series.

Standard 2 — Geometry and Measurement

Students relate geometry to algebra by using coordinate geometry to determine congruence, similarity, symmetry, and tessellations. They prove that triangles are congruent or similar. Students find the equation of a circle in the coordinate plane and describe and make regular and nonregular polyhedra (cube, pyramid, tetrahedron, octahedron, etc.). They understand the properties of congruent and similar solids.

Standard 3 — Data Analysis and Statistics

Students will apply basic ideas related to surveys, construct simulated sampling distributions, interpret margin of error and confidence intervals, and understand standard deviation.

Standard 4 — Probability

Students will apply the Addition Rule for mutually exclusive events.

Standard 5 — Discrete Mathematics

Students will use iteration and recursion to solve problems.

Standard 6 — Trigonometry

Students will use the Law of Sines and Law of Cosines to find measures of sides and angles in triangles. They will also analyze families of trigonometric functions.

Standard 7 — Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to justifying the steps in simplifying functions and solving equations and to deciding whether algebraic statements are true. Students also apply these skills to constructing logical arguments and learn about inductive and deductive reasoning, as well as how to use counterexamples to show that a general statement is false.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Algebra and Functions

Students solve inequalities, quadratic equations, and systems of equations. They graph polynomial, rational, algebraic, and piece-wise defined functions. They graph and write the equations of conic sections and compute with and factor polynomials and algebraic fractions. They solve problems involving exponential and logarithmic expressions, as well as define and use arithmetic and geometric sequences and series.

IM3.1.1 Solve combined linear inequalities.

Example: Solve the inequalities $-7 < 3x + 5 < 11$.

IM3.1.2 Use a graph to find the solution set of a pair of linear inequalities in two variables.

Example: Graph the inequalities $y \leq 4$ and $x + y \leq 5$. Shade the region where both inequalities are true.

IM3.1.3 Find a common monomial factor in a polynomial.

Example: Factor $36xy^2 + 18xy^4 - 12x^2y^4$.

IM3.1.4 Factor the difference of two squares and other quadratics.

Example: Factor $4x^2 - 25$ and $2x^2 - 7x + 3$.

IM3.1.5 Simplify algebraic ratios.

Example: Simplify $\frac{x^2 - 16}{x^2 + 4x}$.

IM3.1.6 Solve algebraic proportions.

Example: Create a tutorial to be posted to the school's Web site to instruct beginning students in the steps involved in solving an algebraic proportion. Use $\frac{x + 5}{4} = \frac{3x + 5}{7}$ as an example.

IM3.1.7 Solve quadratic equations by factoring.

Example: Solve the equation $x^2 - 3x + 2 = 0$ by factoring.

IM3.1.8 Solve quadratic equations in which a perfect square equals a constant.

Example: Solve the equation $(x - 7)^2 = 64$.

IM3.1.9 Complete the square to solve quadratic equations.

Example: Solve the equation $x^2 - 7x + 9 = 0$ by completing the square.

IM3.1.10 Derive the quadratic formula by completing the square.

Example: Prove that the equation $ax^2 + bx + c = 0$ has solutions $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

IM3.1.11 Solve equations that contain radical expressions.

Example: Solve the equation $\sqrt{x + 6} = x$.

IM3.1.12 Recognize and graph various types of functions, including polynomial, rational, and algebraic functions.

Example: Draw the graphs of the functions $y = x^4 - x^2$, $y = \frac{7}{x - 2}$, and $y = \sqrt{x + 2}$.

IM3.1.13 Use function notation. Add, subtract, multiply, and divide pairs of functions.

Example: Let $f(x) = 7x + 2$ and $g(x) = x^2$. Find the value of $f(x) \bullet g(x)$.

IM3.1.14 Understand composition of functions and combine functions by composition.

Example: Let $f(x) = x^3$ and $g(x) = x - 2$. Find $f(g(x))$.

IM3

IM3.1.15 **Graph relations and functions with and without graphing technology.**

Example: Draw the graph of $y = x^3 - 3x^2 - x + 3$.

IM3.1.16 **Find the zeros of a function.**

Example: In the example in indicator 1.15, find the zeros of the function; i.e., find x when $y = 0$.

IM3.1.17 **Solve an inequality by examining the graph.**

Example: Find the solution for $x^3 - 3x^2 - x + 3 < 0$ by graphing $y = x^3 - 3x^2 - x + 3$.

IM3.1.18 **Graph functions defined piece-wise.**

Example: Sketch the graph of $f(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x > 0 \end{cases}$.

IM3.1.19 **Graph absolute value equations and inequalities.**

Example: Draw the graph of $y = 2x - 5$ and use that graph to draw the graph of $y = |2x - 5|$.

IM3.1.20 **Use substitution, elimination, and matrices to solve systems of two or three equations in two or three variables.**

Example: Solve the system of equations: $x - 2y + 3z = 5, x + 3z = 11, 5y - 6z = 9$.

IM3.1.21 **Use systems of linear equations and inequalities to solve word problems.**

Example: Each week you can work no more than 20 hours all together at the local bookstore and the drugstore. You prefer the bookstore and want to work at least 10 more hours there than at the drugstore. Draw a graph to show the possible combinations of hours that you could work.

IM3.1.22 **Define complex numbers and perform basic operations with them.**

Example: Multiply $7 - 4i$ and $10 + 6i$.

IM3.1.23 **Understand how real and complex numbers are related, including plotting complex numbers as points in the plane.**

Example: Plot the points corresponding to $3 - 2i$ and $1 + 4i$. Add these complex numbers and plot the result. How is this point related to the other two?

IM3.1.24 **Solve quadratic equations in the complex number system.**

Example: Solve $x^2 - 2x + 5 = 0$ over the complex numbers.

IM3.1.25 **Solve word problems using quadratic equations.**

Example: You have 100 feet of fencing to make three sides of a rectangular area using an existing straight fence as the fourth side. Construct a formula in a spreadsheet to determine the area you can enclose and use the spreadsheet to make a conjecture about the maximum area possible. Prove (or disprove) your conjecture by solving an appropriate quadratic equation.

IM3.1.26 **Solve equations that contain radical expressions.**

Example: Solve the equation $\sqrt{x + 9} = 9 - \sqrt{x}$.

IM3.1.27 **Solve pairs of equations, one quadratic and one linear or both quadratic.**

Example: Solve the system of equations $y = x^2 - 5x + 1, x + y + 2 = 0$.

IM3.1.28 Write the equations of conic sections (circle, ellipse, parabola, and hyperbola).

Example: Write an equation for a parabola with focus $(2, 3)$ and directrix $y = 1$.

IM3.1.29 Graph conic sections.

Example: Graph the circle described by the equation $(x + 4)^2 + (y - 1)^2 = 9$.

IM3.1.30 Understand the binomial theorem and use it to expand binomial expressions raised to positive integer powers.

Example: Expand $(x + 2)^4$.

IM3.1.31 Divide polynomials by others of lower degree.

Example: Divide $2x^3 - 3x^2 + x - 6$ by $x^2 + 2$.

IM3.1.32 Factor polynomials completely and solve polynomial equations by factoring.

Example: Solve $x^3 + 27 = 0$ by factoring.

IM3.1.33 Use graphing technology to find approximate solutions for polynomial equations.

Example: Approximate the solution(s) of $x^4 - 3x^3 + 2x - 7 = 0$ to the nearest tenth.

IM3.1.34 Use polynomial equations to solve word problems.

Example: You want to make an open-top box with a volume of 500 square inches from a piece of cardboard that is 25 inches by 15 inches by cutting squares from the corners and folding up the sides. Find the possible dimensions of the box.

IM3.1.35 Write a polynomial equation given its solutions.

Example: Write an equation that has solutions $x = 2, x = 5i$ and $x = -5i$.

IM3.1.36 Understand and describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial expression.

Example: Solve the equation $x^4 + x^3 - 7x^2 - x + 6 = 0$, given that $x - 2$ and $x + 3$ are factors of $x^4 + x^3 - 7x^2 - x + 6$.

IM3.1.37 Understand and use negative and fractional exponents.

Example: Simplify $(2a^{-2}b^3)^4 (4a^3b^{-1})^{-2}$.

IM3.1.38 Add, subtract, multiply, divide, and simplify algebraic fractions.

Example: Simplify $\frac{x^2 - 4}{x^5} \div \frac{x^3 - 8}{x^8}$.

IM3.1.39 Simplify complex fractions.

Example: Simplify $\left(\frac{5}{x - 2} + \frac{2}{x + 3}\right) \div \left(\frac{1}{x + 3} + \frac{7}{x - 2}\right)$.

IM3.1.40 Solve equations involving algebraic fractions.

Example: Solve $\frac{10}{n} + \frac{5}{n^2 - 4} = \frac{7}{n - 2}$.

IM3.1.41 Solve word problems involving fractional equations.

Example: Two students, working independently, can complete a particular job in 20 minutes and 30 minutes, respectively. How long will it take to complete the job if they work together?

IM3.1.42 Solve problems of direct, inverse, and joint variation.

Example: One day your drive to work takes 10 minutes and you average 30 mph. The next day the drive takes 15 minutes. What is your average speed that day?

IM3.1.43 Prove simple laws of logarithms.

Example: Use the fact that $a^x \bullet a^y = a^{x+y}$ to show that $\log_a(pq) = \log_a p + \log_a q$.

IM3.1.44 Understand and use the inverse relationship between exponents and logarithms.

Example: Find the value of $\log_{10}(10^7)$.

IM3.1.45 Solve logarithmic and exponential equations and inequalities.

Example: Solve the equation $\log_2 x = 5$.

IM3.1.46 Use the definition of logarithms to convert logarithms from one base to another.

Example: Write $\log_{10} 75$ as a logarithm to base 2.

IM3.1.47 Use the properties of logarithms to simplify logarithmic expressions and to find their approximate values.

Example: Simplify $\log_3 81$.

IM3.1.48 Use calculators to find decimal approximations of natural and common logarithmic numeric expressions.

Example: Find a decimal approximation for ln 500.

IM3.1.49 Solve word problems involving applications of exponential functions to growth and decay.

Example: The population of a certain country can be modeled by the equation $P(t) = 50e^{0.02t}$, where P is the population in millions and t is the number of years after 1900. Find when the population is 100 million, 200 million, and 400 million. What do you notice about these time periods?

IM3.1.50 Define arithmetic and geometric sequences and series.

Example: What type of sequence is 10, 100, 1,000, 10,000, …?

IM3.1.51 Find specified terms of arithmetic and geometric sequences.

Example: Find the tenth term of the arithmetic sequence 3, 7, 11, 15, … .

IM3.1.52 Find partial sums of arithmetic and geometric series.

Example: In the example in 1.51, find the sum of the first 10 terms.

IM3.1.53 Solve word problems involving applications of sequences and series.

Example: You have on a Petri dish 1 square millimeter of a mold that doubles in size each day. What area will it cover after a month?

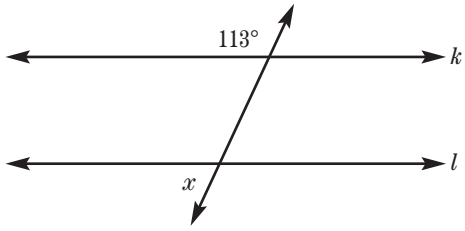
Standard 2

Geometry and Measurement

Students describe and use parallel and perpendicular lines. They use coordinate geometry and prove that triangles are congruent or similar. They find the equation of a circle in the coordinate plane and describe and use properties of solids.

IM3.2.1 Understand and use the relationships between special pairs of angles formed by parallel lines and transversals.

Example: In the diagram, the lines k and l are parallel. What is the measure of angle x? Explain your answer.



IM3.2.2 Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.

Example: Find an equation of a line perpendicular to $y = 4x - 2$.

IM3.2.3 Use properties of congruent and similar polygons to solve problems.

Example: Divide a regular hexagon into triangles by joining the center to each vertex. Show that these triangles are all the same size and shape and find the sizes of the interior angles of the hexagon.

IM3.2.4 Use coordinate geometry to prove properties of polygons such as regularity, congruence, and similarity.

Example: Is the polygon formed by connecting the points (2, 1), (6, 2), (5, 6), and (1, 5) a square?

IM3.2.5 Describe, classify, and understand relationships among the quadrilaterals square, rectangles, rhombus, parallelogram, trapezoid, and kite.

Example: Use a drawing program to create a square, rectangle, rhombus, parallelogram, trapezoid, and kite. Judge which of the quadrilaterals has perpendicular diagonals and draw those diagonals in the figures. Give a convincing argument that your judgment is correct.

IM3.2.6 Use coordinate geometry to prove properties of quadrilaterals such as regularity, congruence, and similarity.

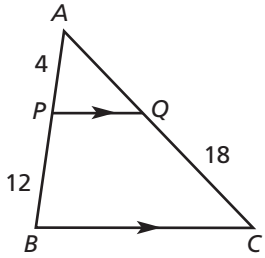
Example: Is rectangle *ABCD* with vertices at (0, 0), (4, 0), (4, 2), (0, 2) congruent to rectangle *PQRS* with vertices at (-2, -1), (2, -1), (2, 1), (-2, 1)?

IM3.2.7 Construct triangles congruent to given triangles.

Example: Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.

IM3.2.8 Prove and apply theorems involving segments divided proportionally.

Example: In triangle ABC, $\overline{PQ}$ is parallel to $\overline{BC}$. What is the length of $\overline{AQ}$?



IM3.2.9 Prove that triangles are congruent or similar and use the concept of corresponding parts of congruent triangles.

Example: In the example in indicator 2.8, prove that triangles ABC and APQ are similar.

IM3.2.10 Use coordinate geometry to prove properties of triangles such as regularity, congruence, and similarity.

Example: Draw a triangle with vertices at (1, 3), (2, 5), and (6, 1). Draw another triangle with vertices at (-3, -1), (-2, 1), and (2, -3). Are these triangles the same shape and size?

IM3.2.11 Find the equation of a circle in the coordinate plane in terms of its center and radius.

Example: Find the equation of the circle with radius 10 and center (6, -3).

IM3.2.12 Describe and make regular and nonregular polyhedra.

Example: Is a cube a regular polyhedron? Explain why or why not.

IM3.2.13 Describe the polyhedron that can be made from a given net (or pattern). Describe the net for a given polygon.

Example: Make a net for a tetrahedron out of poster board and fold it up to make the tetrahedron.

IM3.2.14 Identify and know properties of congruent and similar solids.

Example: Explain how the surface area and volume of similar cylinders are related.

IM3.2.15 Find and use measures of sides, volumes of solids, and surface areas of solids. Relate these measures to each other using formulas.

Example: An ice cube is dropped into a glass that is roughly a right cylinder with a 6 cm diameter. The water level rises 1 mm. What is the volume of the ice cube?

Standard 3

Data Analysis and Statistics

Students design and interpret surveys, use sampling distributions, and understand standard deviation.

IM3.3.1 Understand and apply basic ideas related to the design and interpretation of surveys, such as background information, random sampling, and bias.

Example: Explain how to design a survey that is random and not biased in nature.

IM3.3.2 Construct simulated sampling distributions of sample proportions and use sampling distributions to identify which proportions are likely to be found in a sample of a given size.

Example: About 30 percent of the students at a school are on the honor roll. If you took a random sample of 30 students, what range of students would be likely to be on the honor roll?

IM3.3.3 Construct and interpret margin of error and confidence intervals for population proportions.

Example: In a random sample of 40 people at a restaurant, 13 of them say that they prefer black coffee. Use a 90 percent confidence interval to find approximately what percent of all patrons at the restaurant prefer black coffee.

IM3.3.4 Understand the standard deviation as a measure of variability in a distribution.

Example: Explain how the values vary about the mean. Given a set of test scores: 99, 96, 94, 93, 90, 88, 86, 77, 70, 68, find the mean and standard deviation.

Standard 4

Probability

Students apply the Addition Rule for mutually exclusive events.

IM3.4.1 Understand and apply the Addition Rule for mutually exclusive events.

Example: Nicki is an 80 percent free-throw shooter. She is in a two-shot foul situation. Find the probability she will make only one free throw.

Standard 5

Discrete Mathematics

Students use iteration and recursion to solve problems.

IM3.5.1 Use iteration and recursion as tools to represent, analyze, and solve problems involving sequential change.

Example: There are 2,500 fish in a pond. Each year the population decreases by 25 percent, but 1,000 fish are added to the pond at the end of the year. Find the population in five years. Also, find the long-term population.

IM3.5.2 Explore function iteration and, in the process, informally introduce function composition.

Example: Iterate the function $f(x) = 3x + 2$ starting with $x = 1$. Find the first four iterates.

IM3.5.3 Understand and apply recursion equations, particularly combined recursion equations of the form $A_n = rA_{n-1} + b$.

Example: Write a recursion equation to model the following situation: You buy a $10,000 car with an annual interest rate of 6 percent and make a monthly payment of $250. Find the number of months needed to pay off the car.

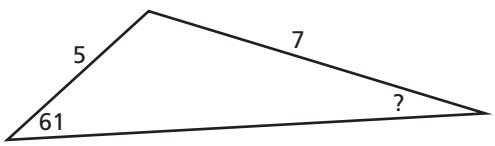
Standard 6

Trigonometry

Students use the Law of Sines and the Law of Cosines to solve problems. They analyze families of trigonometric functions.

IM3.6.1 Find the measures of sides and angles in triangles using the Law of Sines.

Example: Find the measure of the missing angle shown.



IM3.6.2 Find the measures of sides and angles in triangles using the Law of Cosines.

Example: In triangle ABC, find the length of side c if side $a = 10$, $b = 26$, and $\angle C = 96°$.

IM3.6.3 Compare and contrast families of trigonometric functions.

Example: Draw a graph of $f(x) = \sin x$, and $f(x) = \cos x$, what are the similarities and differences of the two graphs?

Standard 7

Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems and develop and evaluate mathematical arguments and proofs.

IM3.7.1 Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation and that the steps taken when solving equations create new equations that have, in most cases, the same solution set as the original. Understand that similar logic applies to solving systems of equations simultaneously.

Example: A student solving the equation $\sqrt{(x+6)} = x$ comes up with the solution set $\{-2, 3\}$. Explain why $\{-2, 3\}$ is not the solution set to this equation, and why the "check" step is essential in solving the equation.

IM3.7.2 Decide whether a given algebraic statement is true always, sometimes, or never (statements involving rational or radical expressions or logarithmic or exponential functions).

Example: Is the statement $(a^x)^y = a^{xy}$ true for all x, for some x, or for no x? Explain your answer.

IM3.7.3 Distinguish between inductive and deductive reasoning, identifying and providing examples of each.

Example: What type of reasoning are you using when you look for a pattern?

IM3.7.4 Identify the hypothesis and conclusion in a logical deduction.

Example: What is the hypothesis and conclusion in this argument: If there is a number x such that $2x + 1 = 7$, then $x = 3$?

IM3.7.5 Use counterexamples to show that statements are false, recognizing that a single counterexample is sufficient to prove a general statement false.

Example: Show by an example that this statement is false: The product of two complex numbers is never a real number.

IM3.7.6 Use the properties of number systems and the order of operations to justify the steps of simplifying functions and solving equations.

Example: Simplify $2(x^3 - 3x^2 + x - 6) - (x - 3)(x + 4)$, explaining why you can take each step.

IM3.7.7 Identify and give examples of undefined terms, axioms, and theorems, and inductive and deductive proofs.

Example: Do you prove axioms from theorems or theorems from axioms?

IM3.7.8 Construct logical arguments, judge their validity, and give counterexamples to disprove statements.

Example: Find an example to show that triangles with two sides and one angle equal are not necessarily congruent.

NOTES